2012 P2 Q2.

(i) $\text{LHS} = \dfrac{4r^2+1}{(2r-1)(2r+1)}$.

$= \dfrac{4r^2-1}{(2r-1)(2r+1)} + \dfrac{1}{(2r-1)(2r+1)}$

$= 1 + \dfrac{1}{2r-1} - \dfrac{1}{2r+1}$. $= \text{RHS}$ //.

(ii) $\displaystyle\sum_{r=1}^{n} \frac{4r^2+1}{(2r-1)(2r+1)}$.

$= \displaystyle\sum_{r=1}^{n} \left[1 + \frac{1}{2r-1} - \frac{1}{2r+1}\right]$

$= \displaystyle\sum_{r=1}^{n} 1 + \sum_{r=1}^{n} \left(\frac{1}{2r-1} - \frac{1}{2r+1}\right)$

$= n + \left(\dfrac{1}{1} - \dfrac{1}{2n+1}\right)$

$= n + 1 - \dfrac{1}{2n+1}$ //

show the workings of method of difference on your own.

(iii) $\displaystyle\sum_{r=5}^{n} \frac{4r^2-8r+5}{(2r-3)(2r-1)}$

replace r by $r+1$.

$$= \sum_{r=4}^{n-1} \frac{4r^2+1}{(2r-1)(2r+1)}$$

$$= \sum_{r=1}^{n-1} \frac{4r^2+1}{(2r-1)(2r+1)} - \sum_{r=1}^{3} \frac{4r^2+1}{(2r-1)(2r+1)}$$

$$= (n-1)+1 - \frac{1}{2(n-1)+1} - \left[3+1 - \frac{1}{2(3)+1}\right]$$

$$= n - \frac{1}{2n-1} - 4 + \frac{1}{7}$$

$$= n - \frac{1}{2n-1} - \frac{27}{7}. \quad //$$