2012 P1 Q10.

(i) $\underset{\sim}{b} = \begin{pmatrix} 6 \\ 4 \\ 0 \end{pmatrix}$, $\underset{\sim}{c} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix}$, $\underset{\sim}{t} = \begin{pmatrix} 3 \\ 2 \\ a \end{pmatrix}$

$\Rightarrow \vec{OR} = \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix}$

$\vec{TR} = \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ a \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 6-a \end{pmatrix}$

$\vec{RS} \parallel \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$\underset{\sim}{n} = \begin{pmatrix} 3 \\ 2 \\ 6-a \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -6+a \\ 2 \end{pmatrix}$

$\pi_{RST}: \underset{\sim}{r} \cdot \begin{pmatrix} 0 \\ -6+a \\ 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -6+a \\ 2 \end{pmatrix}$

$(-6+a)y + 2z = -12 + 4a$

$(-6+a)y + 2z - 4a + 12 = 0.$ (shown).

(ii) $\theta = \measuredangle$ b/w both the nor

$$\cos\theta = \frac{\left| \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -6+a \\ 2 \end{pmatrix} \right|}{\sqrt{1}\ \sqrt{(-6+a)^2 + 2^2}} = \frac{1}{\sqrt{2}}.$$

$$\frac{|1-6+a|}{\sqrt{40-12a+a^2}} = \frac{1}{\sqrt{2}}.$$

$$a = 8 \quad \text{or} \quad 4 \text{ (rej)}.$$

$\therefore$ $a=8$ since T is above P.

(iii) $\overrightarrow{WR} = \begin{pmatrix} 6 \\ 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$

$l_{WR}: \underset{\sim}{r} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \lambda \in \mathbb{R}$

$\overrightarrow{MN} = \begin{pmatrix} 6 \\ 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$

$\underset{\sim}{n} = \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 0 \end{pmatrix}$

$\pi_{screen}: \underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$

$\Rightarrow \underset{\sim}{r} \cdot \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix} = 15.$

let $\overrightarrow{OV} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}$ for some λ.

$$\left(\begin{pmatrix}3\\3\\2\end{pmatrix} + \lambda\begin{pmatrix}3\\1\\4\end{pmatrix}\right)\cdot\begin{pmatrix}1\\3\\0\end{pmatrix} = 15$$

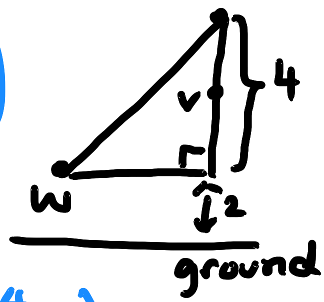
$$\lambda = \frac{1}{2}.$$

$$\Rightarrow \quad \vec{OV} = \begin{pmatrix}3\\3\\2\end{pmatrix} + \left(\frac{1}{2}\right)\begin{pmatrix}3\\1\\4\end{pmatrix} = \left(\frac{9}{2}, \frac{7}{2}, 4\right).$$

(iv) Beam hits $\left(\frac{9}{2}, \frac{7}{2}, 6\right)$ on screen

to make max. angle

$$\begin{pmatrix}9/2\\7/2\\6\end{pmatrix} - \begin{pmatrix}3\\3\\2\end{pmatrix} = \begin{pmatrix}3/2\\1/2\\4\end{pmatrix}$$



$$\left|\begin{pmatrix}3/2\\1/2\\4\end{pmatrix}\right| = \sqrt{\frac{37}{2}}.$$

$$\text{required angle} = \cos^{-1}\left(\frac{4}{\sqrt{\frac{37}{2}}}\right)$$

$$= 21.56 \approx 21.6^\circ.$$

(v) screen : $\underset{\sim}{r} \cdot \begin{pmatrix}1\\3\\0\end{pmatrix} = 15$

$$\Rightarrow \underset{\sim}{r} \cdot \frac{\begin{pmatrix}1\\3\\0\end{pmatrix}}{\left|\begin{pmatrix}1\\3\\0\end{pmatrix}\right|} = \frac{15}{\left|\begin{pmatrix}1\\3\\0\end{pmatrix}\right|}$$

$$\underset{\sim}{r} \cdot \frac{\begin{pmatrix}1\\3\\0\end{pmatrix}}{\sqrt{10}} = \frac{15}{\sqrt{10}}$$

For it to be $\sqrt{2}$ units closer to O,

$$\Rightarrow \underset{\sim}{r} \cdot \frac{\begin{pmatrix}1\\3\\0\end{pmatrix}}{\sqrt{10}} = \frac{15}{\sqrt{10}} - \sqrt{2}$$

$$\Rightarrow x + 3y = 15 - 2\sqrt{5}. \quad //$$