Class Adm No

Candidate Name: ______________________________

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millennia institute

2016 Preliminary Examination II
Pre-University 3

MATHEMATICS **9740/01**

Paper 1 **13 September 2016**

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

1 Without using a calculator, solve the inequality $\frac{2x^2+2x-3}{x^2-5} \geq 1$. [4]

2 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ such that

$$\mathbf{a} = \mathbf{i} + 5p\mathbf{j} + 2p\mathbf{k} \quad \text{and} \quad \mathbf{b} = -12\mathbf{i} + 3\mathbf{j} + 4\mathbf{k},$$

where $p > 0$. Find the exact value of p if

(a) $\mathbf{a}$ and $\mathbf{b}$ are perpendicular vectors; [2]

(b) AOB is an isosceles triangle where $\angle OAB = \angle OBA$. [2]

3 Points A and B have coordinates $(3, 1, 2)$ and $(1, -5, 4)$. The point C lies on AB produced such that $AC : BC = 3 : 2$. Find the position vector of point C. [2]

Determine the position vector of point D such that $OADC$ is a parallelogram, and find the exact area of $OADC$. [4]

4 The complex number a is given by $p + q\text{i}$, where p and q are positive real numbers. The complex number z satisfies the relations

$$|z - a| = |a| \quad \text{and} \quad -\frac{\pi}{2} \leq \arg(z - a) \leq \frac{\pi}{2}.$$

(i) On an Argand diagram, sketch the region in which the point representing z can lie. [4]

(ii) Hence find, in terms of p and q, the value of z for which

(a) the imaginary part of z is zero; [1]

(b) $\arg z$ is maximum. [2]

5 **(a)** Find $\int \frac{2x+3}{x^2-9}\, dx$. [3]

(b) Use the substitution $t = \tan\frac{1}{2}\theta$ to find $\int \frac{1}{1+\cos\theta}\, d\theta$. [4]

6 [It is given that a sphere of radius r has surface area $4\pi r^2$ and volume $\frac{4}{3}\pi r^3$.]

An architect designs a model of a theatre, which consists of a hemisphere of radius r cm joined directly on top of a cylinder of radius r cm and height h cm, as well as a circular floor of radius r cm. It is given that the total surface area of the model is a fixed value a cm^2.

(i) Show that the volume of the model is given by $V = \frac{1}{2}ar - \frac{5}{6}\pi r^3$. [3]

(ii) Use differentiation to find the value of r such that V is maximum. Show also that h and r are equal when V is maximum. [4]

(iii) What is the assumption needed for your calculations in parts **(i)** and **(ii)** to be valid? [1]

7 The region R is bounded by the curve $y = \ln x$, the line $x = 5$ and the x-axis.

(i) Find the exact area of R, showing your working clearly. [3]

(ii) Find the volume of revolution when R is rotated completely about the x-axis, giving your answer to 3 decimal places. [2]

(iii) Use a non-calculator method to find the volume of revolution when R is rotated completely about the y-axis, simplifying your answer. [4]

8 Tunnel boring machines are used to build underground tunnels for new rail lines by cutting through soil.

(a) Using machine A, the distance cut on the first day is 1.5 metres. On each subsequent day, the difference in the distance cut compared to the previous day is d metres.

(i) Machine A cuts 1.43 metres on the 6^{th} day. Find the value of d. [1]

(ii) Write down an expression for the length of the tunnel produced by machine A after n days. Hence find the least number of days that machine A needs to produce a tunnel of 30 metres. [3]

(b) Using machine B, the distance cut on the first day is also 1.5 metres. On each subsequent day, the distance cut is 0.99 of the distance cut on the previous day.

(i) Find the theoretical maximum length of the tunnel produced by machine B.

[1]

(ii) Write down an expression for the length of the tunnel produced by machine B after n days. [1]

(iii) Machine A and machine B are both used to cut one tunnel each. What is the least number of days for one machine to produce more than 1 metre of tunnel than the other machine? [2]

9 A sequence $u_1, u_2, u_3, \ldots$ is given by

$$u_1 = 1 \quad \text{and} \quad u_{n+1} = u_n - \frac{3n^2+3n+1}{n^3(n+1)^3} \text{ for } n \geq 1.$$

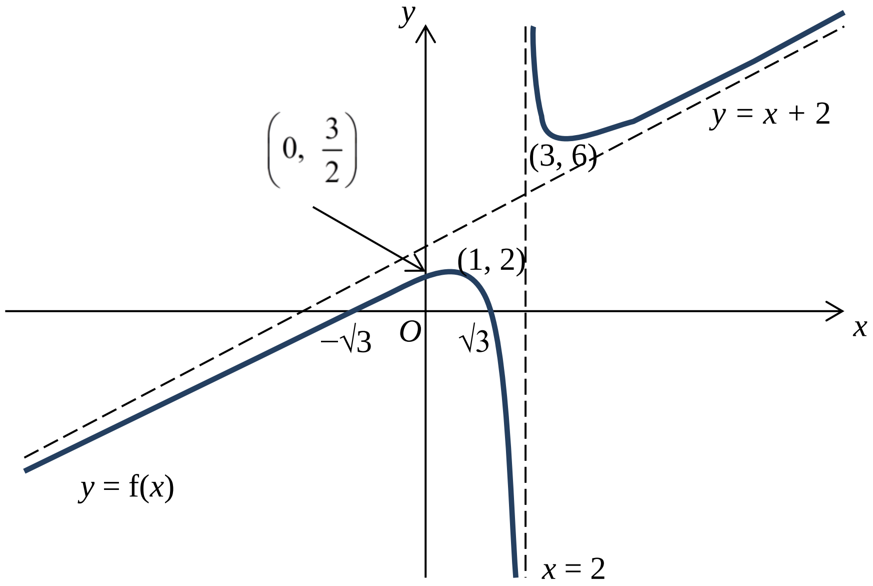
(i) Use the method of mathematical induction to prove that $u_n = \dfrac{1}{n^3}$. [4]

(ii) Hence find $\displaystyle\sum_{n=1}^{N} \frac{3n^2+3n+1}{n^3(n+1)^3}$. [2]

(iii) Give a reason why the series in part **(ii)** converges, and write down the value of the sum to infinity. [2]

(iv) Use your answer to part **(ii)** to find $\displaystyle\sum_{n=4}^{N+2} \frac{3(n-1)^2+3(n-1)+1}{(n-1)^3 n^3}$. [3]

10 **(a)** The curve $y = \mathrm{f}(x)$ has a maximum point at $(1,\ 2)$ and a minimum point at $(3,\ 6)$, and crosses the axes at $(-\sqrt{3},\ 0)$, $(\sqrt{3},\ 0)$ and $\left(0,\ \dfrac{3}{2}\right)$. It has asymptotes $y = x + 2$ and $x = 2$.



State the range of values of x for which the curve $y = \mathrm{f}(x)$ is decreasing. [1]

On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}(|x|)$; [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$; [3]

labelling clearly the equations of any asymptotes, as well as the coordinates of any axial intercepts and turning points.

(b) The curve C has equation $x^2 + 4y^2 = 4$.

(i) Sketch C, stating clearly the coordinates of any point(s) of intersections with the axes. [2]

(ii) Write down the range of values of k for which the equation $x^2 + 4(kx+1)^2 = 4$ has two distinct roots. [1]

(iii) Describe fully a sequence of transformations which would transform the curve C into a circle with centre $(1,\ 0)$ and radius 1. [2]

11 A curve C has parametric equations

$$x = \sin^2\theta, \qquad y = 2\sin\theta - \sin^3\theta,$$

where $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$.

(i) Find the value(s) of the parameter θ at the point(s) where the curve C is stationary. [4]

(ii) Show that the equation of the tangent to the curve at the point where the gradient is $\frac{5}{4}$ is given by $y = \frac{5}{4}x + \frac{9}{16}$. [4]

(iii) Determine whether the tangent $y = \frac{5}{4}x + \frac{9}{16}$ meets the curve C again. [3]

12 **(a)** **(i)** One root of the equation $az^3 - 9z^2 + bz - 5 = 0$ is $z = 2 - \mathrm{i}$. Find the values of a and b. [4]

(ii) Hence find all the roots of the equation in part **(a)(i)** in exact form. [4]

(b) Solve the equation $w^5 = 32$, giving the roots in the form $r\mathrm{e}^{\mathrm{i}\alpha}$, where $r > 0$ and $-\pi < \alpha \le \pi$. [3]

When shown on an Argand diagram, these roots form the vertices of a polygon. Deduce the area of this polygon in the form $k\sin\theta$, where k and θ are values to be determined. [2]

End of Paper