NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS **9740**

Paper 1 **15th September 2**

3 Ho

Additional Materials: Cover Sheet
Answer Papers
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

1 The *n*th term of a sequence is given by $u_n = \dfrac{4^n n^2}{(n+1)(n+2)}$, for $n \geq 1$.

The sum of the first *n* terms is denoted by S_n. Use the method of mathematical induction to show

2 Using partial fractions, find $\displaystyle\int_{-2}^{2} \frac{17x^2 + 23x + 12}{(3x+4)(x^2+4)}\, \mathrm{d}x$, leaving your answer in exact form.

3 A curve *C* has parametric equations

$$x = \frac{1}{2}(\sin t \cos t + t), \quad y = \frac{1}{2}t - \frac{1}{4}\sin 2t, \quad \text{for } -\frac{\pi}{2} < t \leq 0.$$

The tangent to the curve at the point *P* has gradient 1. Find the equation of the normal at *P*.

The region bounded by this normal, the curve *C* and the *x*-axis is rotated through 2π radians about the *x*-axis. Find, to 5 decimal places, the volume of the solid obtained.

What can be said about the tangents to the curve as *t* approaches 0? **[7]**

4 Referred to the origin, the points *A* and *B* have position vectors **a** and **b** respectively.

A point *C* is such that *OACB* forms a parallelogram. Given that *M* is the mid-point of *AC*, find th

Point *P* is on *AB* is such that *MP* is perpendicular to *AB*. Given that angle *AOB* is 60^o, $|\mathbf{a}| = 2$ and

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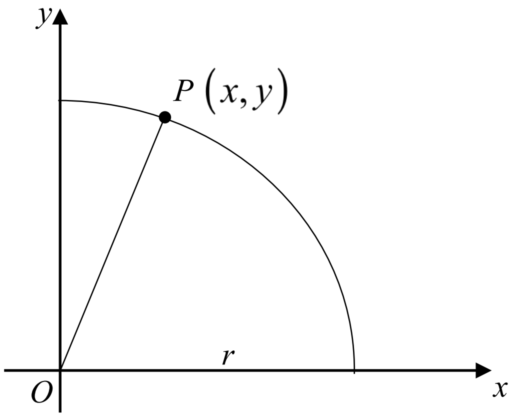

A particle P moves along the curve with equation $x^2 + y^2 = r^2$, where $x \ge 0, y \ge 0,$ and r is a cons

Given that the rate of change of y with respect to time t is 0.1% of r, show that $\frac{\mathrm{d}m}{\mathrm{d}t} = \left(\frac{r}{10\sqrt{r^2 - y^2}} \right)^3$.

State the geometrical meaning of $\frac{\mathrm{d}m}{\mathrm{d}t}$. **[7]**

6 **(i)** Show that $\frac{r^2 + r - 1}{(r+2)!} = \frac{A}{r!} + \frac{B}{(r+1)!} + \frac{C}{(r+2)!}$, where A, B and C are constants

(ii) Hence find $\sum_{r=1}^{n} \frac{r^2 + r - 1}{(r+2)!}$ in terms of n. (There is no need to express your answer as a single algebraic function.) **[3]**

(iii) Explain why $\sum_{r=1}^{n} \frac{r^2 - 1}{(r+2)!} < \frac{1}{2}$. **[2]**

(iv) Use your answer to part **(ii)** to find $\sum_{r=4}^{n} \frac{r^2 - 3r + 1}{r!}$ in terms of n. **[3]**

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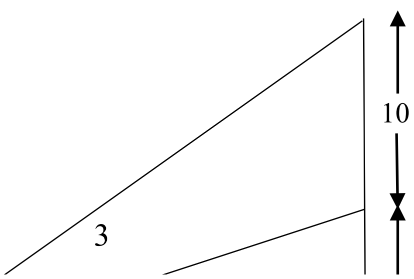

A 10 feet tall statue is mounted on a 12 feet tall pedestal. A boy is standing x feet away from the pedestal. His eyes are 5 feet above ground level, and the angle subtended by the statue from the boy's eyes is θ radians (see diagram).

Prove that

$$\tan\theta = \frac{10x}{119 + x^2} .$$

Hence, or otherwise, find the exact value of x for which θ is maximum and justify that this value of x gives the maximum value of θ.

Deduce, to the nearest degree, the maximum angle subtended by the statue from the boy's eyes. **[9]**

8 **(i)** Find the fourth roots of $-1+\sqrt{3}\,\mathrm{i}$, giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi$ ·

(ii) Hence, or otherwise, write down the roots of the equation $(1+z)^4 + 1 - \mathrm{i}\sqrt{3} = 0$ and show the roots Z_i, $i = 1,2,3,4$ on an Argand diagram. **[3]**

(iii) Illustrate, using the same Argand diagram, the locus of a point Q representing the complex number v, where $\left| v + 1 - 4\sqrt{3} - 4\mathrm{i} \right| = 2$.

Hence find the exact greatest and least possible values of Z_iQ . **[4]**

9 Two biologists are investigating the growth of a certain bacteria of size x hundred thousand at time t days. It is known that the number of bacteria initially is 20% of a, where a is a positive constant.

(i) One biologist believes that x and t are related by the differential equation $\frac{\mathrm{d}x}{\mathrm{d}t} = x(a-x)$. Given that the number of bacteria increases to 50% of a when $t = \ln 2$ days, show that $x = \frac{2}{4\mathrm{e}^{-2t}+1}$. **[7]**

(ii) Another biologist believes that x and t are related by the differential equation $\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = 10-9t^2$. Find the general solution of this differential equation and sketch three members of the family of solution curves. **[5]**

10 **(a)** **(i)** Express $\sin x + \sqrt{3}\cos x$ in the form $R\sin(x+\alpha)$ where R and α

The function f is defined by $\mathrm{f}: x \mapsto \sin x + \sqrt{3}\cos x,\ \frac{\pi}{6} \le x \le k.$

(ii) Find the largest exact value of k such that f has an inverse. Hence define f^{-1} in similar form and write down the set of values of x for which $\mathrm{ff}^{-1}(x) = \mathrm{f}^{-1}\mathrm{f}(x)$. **[5]**

(b) The function g is defined by $\mathrm{g}: x \mapsto 2 - \frac{5x}{1+x^2},\ x \in \mathbb{R}$.

(i) Use an algebraic method to find the range of g. **[3]**

(ii) State a sequence of transformations which transform the graph of $y = \mathrm{g}(x)$ to the gra

11 The line l_1 passes through the point A with coordinates (1, 2, 1) and is parallel to the vector $\mathbf{i} + a\mathbf{j} + 2\mathbf{k}$, where $a \in$ ° . The line l_2 has equation $x - 3 = \dfrac{y}{2} = \dfrac{z-5}{3}$. It is given that l_1 and l_2 intersect at point B.

(i) Find the value of a. **[4]**

(ii) The plane p_1 contains the point A and is perpendicular to l_2. Find the exact shortest distance from point B to p_1. Hence find the acute angle between l_1 and p_1. **[5]**

(iii) Find a cartesian equation of plane p_2 that is perpendicular to p_1 and contains l_1. **[3]**

(iv) Find the acute angle between p_2 and the xy-plane. **[2]**