



**JURONG JUNIOR COLLEGE**  
**Preliminary Examinations**

**MATHEMATICS**  
**Higher 2**

**9740 / 2**  
**31 August 2016**  
**3 hours**

Additional materials:      Answer Paper  
   Cover Page  
   List of Formulae (MF 15)

**READ THESE INSTRUCTIONS FIRST**

Write your name and civics class on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a HB pencil for any diagrams or graphs.  
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

**At the end of the examination, fasten all your work securely together, with the cover page in front.**

The number of marks is given in brackets [ ] at the end of each question or part question.

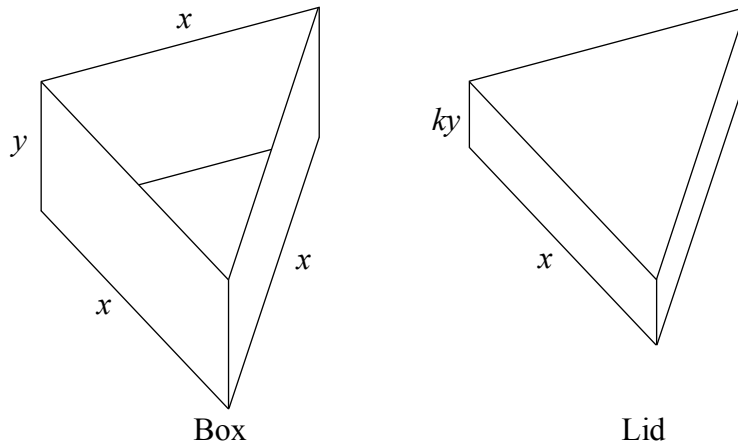
This document consists of 7 printed pages and 1 blank page.

**[Turn over**

## Section A: Pure Mathematics [40 marks]

- 1 (a) Given that  $x$  and  $y$  are related by  $\frac{dy}{dx} = \sec^2 y$  and that  $y = 0$  when  $x = 1$ , find  $x$  in terms of  $y$ . [4]
- (b) A medical researcher is investigating the rate of spread of a virus in a group of people of size  $n$  at time  $t$  weeks. He suggests that  $n$  and  $t$  are related by the differential equation  $\frac{d^2n}{dt^2} = e^{-\frac{t}{5}}$ .
- (i) Find the general solution of the differential equation, giving your answer in the form  $n = f(t)$ . [2]
- (ii) Explain why all solution curves of the differential equation are concave upwards. [1]
- (iii) It is given that initially, the number of people infected with the virus is 50. Sketch on a single diagram, two distinct solution curves for the differential equation to illustrate the following two cases for large values of  $t$ :
- I. the population of infected people increases indefinitely,
- II. the population of infected people stabilizes at a certain positive number. [3]
- 2 (a) A parallelogram has two adjacent sides defined by the vectors  $\mathbf{a}$  and  $2\mathbf{a} + 3\mathbf{b}$ . Given that the magnitudes of  $\mathbf{a}$  and  $\mathbf{b}$  are 4 and 5 respectively and the angle between  $\mathbf{a}$  and  $\mathbf{b}$  is  $30^\circ$ , find the area of the parallelogram. [4]
- (b) A point  $P$  has coordinates  $(2, -1, -2)$  and a line  $l$  has equation  $\frac{x-1}{2} = 1-z, y=3$ .
- (i) Find the perpendicular distance from  $P$  to  $l$ . [4]
- (ii) Find the acute angle between  $l$  and the line  $L$  that is parallel to the  $z$ -axis. [2]

- 3 A box with volume  $250 \text{ cm}^3$  is made of cardboard of negligible thickness. It has a height of  $y$  cm and an equilateral triangular base of side  $x$  cm. Its lid has depth  $ky$  cm, where  $0 < k \leq 1$  (see diagram).



- (i) Show that the total external surface area of the box and lid can be expressed as

$$\frac{1000\sqrt{3}(1+k)}{x} + \frac{\sqrt{3}}{2}x^2. \quad [4]$$

- (ii) Use differentiation to find, in terms of  $k$ , the value of  $x$  that gives a minimum total external surface area of the box and lid. [3]
- (iii) Find the ratio  $\frac{y}{x}$  in this case, in terms of  $k$ , simplifying your answer. [2]
- (iv) Find the values for which  $\frac{y}{x}$  must lie. [2]

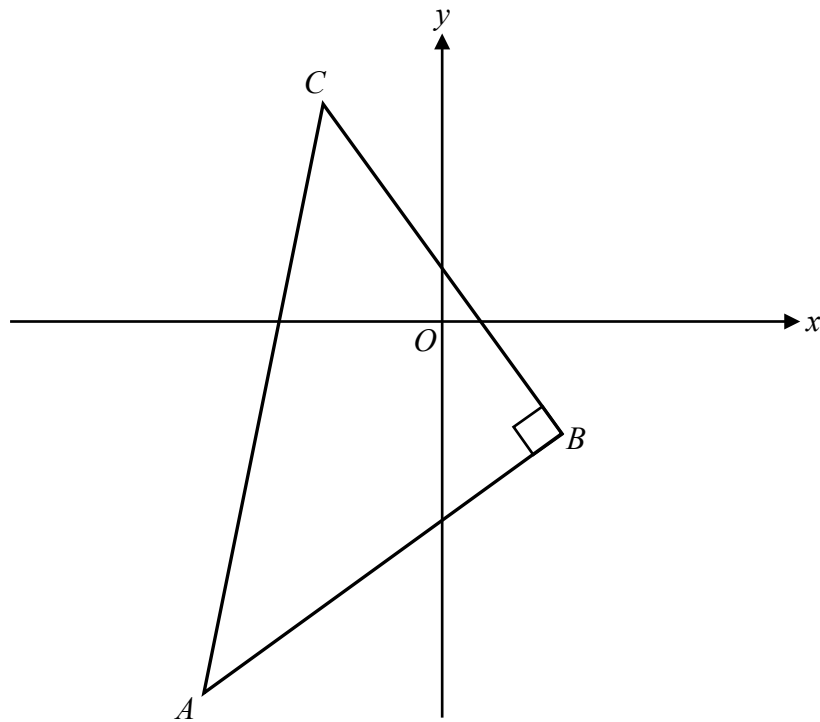
4 The complex numbers  $a$  and  $b$  are given by  $a = -(1 + \sqrt{3}i)$  and  $b = \frac{1}{2}(1 - i)$ .

(i) Without using a calculator, find the value of  $a^2b$  in the form  $x + iy$ . [2]

(ii) By using the moduli and arguments of  $a$  and  $b$ , find the modulus and argument of  $a^2b$ . [3]

(iii) Use your answers to parts (i) and (ii) to show that  $\sin \frac{5\pi}{12} = \frac{\sqrt{3} + 1}{2\sqrt{2}}$ . [2]

(iv) The diagram below shows an isosceles right triangle  $ABC$ , where the points  $A$ ,  $B$  and  $C$  represent the complex numbers  $a$ ,  $b$  and  $c$  respectively. Find the exact value of  $c$ . [2]



**Section B: Statistics [60 marks]**

5 A group of 11 people consists of 6 men and 5 women, 3 of whom are sisters. A committee consisting of six people is to be selected. Find the number of ways the committee can be formed if

- (i) it consists of exactly two men, [1]  
 (ii) it includes at least one of the sisters. [2]

Given that the chosen committee consists of 2 sisters, Sue and Suzy, together with 3 other men, Muthu, Mark, Michael and 1 other woman, Wina. They are seated at a round table meant for six people. Find the number of possible arrangements if

- (iii) one of the men is to be seated between the two sisters, [2]  
 (iv) the two sisters are sitting directly opposite each other. [2]

6 The table below shows the number of male and female students studying Chemistry, Physics and Biology at a private school.

	Chemistry	Physics	Biology
Male	200	130	70
Female	250	300	50

One of the students is chosen at random. Events  $C$ ,  $B$  and  $M$  are defined as follows:

$C$  : The student chosen is studying Chemistry.

$B$  : The student chosen is studying Biology.

$M$  : The student chosen is a male.

Find

- (i)  $P(C | M)$ , [1]  
 (ii)  $P(M \cup C)$ , [1]  
 (iii)  $P(M' \cap B')$ . [1]

Determine whether  $C$  and  $M$  are independent. [2]

It is given that 20% of Chemistry students, 30% of Physics students and 5% of Biology students are international students.

- (iv) One of the students selected at random is an international student. What is the probability that this student studies Chemistry? [2]  
 (v) Three students are chosen at random. Find the probability that there is exactly one international student who studies Physics. [2]

**[Turn over**

- 7 In order to investigate whether there is a correlation between rainfall and crop yields, the total rainfall,  $x$  mm, and the weights of a particular crop per square metre,  $y$  kg, were recorded in a number of fields. The data are shown below.

$x$	36	72	44	74	64	50
$y$	2.2	8.4	1.8	7.4	4.3	2.2

- (i) Draw a scatter diagram to illustrate the data. [2]
- (ii) Calculate the value of the product moment correlation coefficient, and explain why its value does not necessarily mean that the best model for the relationship between  $x$  and  $y$  is  $y = a + bx$ . [2]
- (iii) By comparing the product moment correlation coefficients, explain whether  $y = a + bx$  or  $y = c + dx^2$  is a better model. [2]
- (iv) Using a suitable regression line, estimate the yield of crop per square metre when the total rainfall is 55mm. Comment on the reliability of your estimation. [3]
- 8 It is known that 8% of the population of a large city use a particular web browser called Voyager. A researcher wishes to interview people from the city who use Voyager and selects people at random, one at a time.
- (i) Find the probability that the first person that he finds uses Voyager is the third person selected. [2]
- A random sample of  $n$  people is now selected.
- (ii) State two conditions needed for the number of people in the sample who use Voyager to be well modelled by binomial distribution. [2]
- (iii) Given that  $n = 80$ , use a suitable approximation to find the probability that, fewer than 10 people use Voyager. [3]
- (iv) Find the least value of  $n$  such that the probability of at least 10 people use Voyager is more than 0.2. [3]

- 9 A supermarket sells boxes of a particular brand of biscuits in two flavours, chocolate and strawberry. The mean number of boxes of chocolate biscuits sold in a day is 2.2.
- (i) Find the probability that in a day, no boxes of chocolate biscuits were sold. [1]
- (ii) In a week of 7 days, find the expected number of days that no boxes of chocolate biscuits were sold. [2]

The mean number of boxes of strawberry biscuits sold in a day is denoted by  $\lambda$ .

- (iii) Given that the probability of less than 2 boxes of strawberry biscuits sold in a day is 0.6, write down an equation for the value of  $\lambda$ , and find  $\lambda$  numerically, correct to 1 decimal place. [3]
- (iv) Find the probability that in a week of 7 days, the total number of boxes of chocolate and strawberry biscuits sold exceeds 25 boxes. [2]
- (v) Use a suitable approximation to find the probability that, in a month of 30 days, the number of boxes of chocolate biscuits sold is more than the number of boxes of strawberry biscuits. [4]
- 10 A researcher is running a trial of a new variety of potato. A field contains 20 rows of the new variety of potato plants, with 80 plants in each row. A researcher intends to dig up 8 plants and measure the mass of potatoes produced by each plant.
- (i) Describe how he could choose a systematic sample of 8 plants from a single row of 80 plants and state the advantage of this sampling method. [3]

The researcher claims that the average mass of the new variety of potato is at least 150g. The mass of a new variety of potato is denoted by  $X$  grams. The masses of a random sample of 80 new variety potatoes are summarized by

$$\sum(x-150) = -160, \quad \sum(x-150)^2 = 5520.$$

- (ii) Calculate the unbiased estimates of the population mean and variance. [2]
- (iii) Test at the 1% significance level, whether the researcher's claim is valid. [4]
- (iv) Explain what you understand by the phrase "at the 1% significance level" in the context of this question. [1]

Another random sample of 8 potatoes was chosen with mean mass 148.5g and standard deviation  $k$  g. Find the range of values that  $k$  can take such that at 1% level of significance, this sample would indicate that the researcher's claim is invalid. [3]

**BLANK PAGE**