Yishun Junior College
2016 JC2 PRELIMINARY EXAMINATION

MATHEMATICS **9740/02**

Higher 2

Paper 2 **24 August 2016**

WEDNESDAY 0800h – 1100h

Additional materials :
Answer paper
Graph paper
List of Formulae (MF15)

TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CTG in the spaces provided on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 A sequence $u_1, u_2, u_3,$ ... is given by

$$u_1 = 1 \quad \text{and} \quad u_{n+1} = nu_n + 1 \quad \text{for } n \geq 1.$$

Use the method of induction to prove that

$$u_n = (n-1)!\sum_{r=0}^{n-1}\frac{1}{r!}.$$ [4]

Hence, find the exact value of $\lim_{n\to\infty}\frac{u_n}{(n-1)!}$. [1]

2 Show that $\mathrm{f}(x) = \frac{5x-2}{x(x-1)(x+2)}$ can be expressed as $\frac{A}{x-1}+\frac{B}{x}+\frac{C}{x+2}$, where A, B and C are constants to be determined. [1]

Hence, find $\sum_{r=2}^{n}\mathrm{f}(r)$. (There is no need to express your answer as a single algebraic fraction.) [3]

Explain, with the aid of a sketch of $y = \mathrm{f}(x)$, $x > 1$, why $\sum_{r=2}^{n}\mathrm{f}(r) > \int_2^{n+1}\mathrm{f}(x)\,\mathrm{d}x$ for $n \geq 2$. [2]

3 The parametric equations of a curve C are $x = t - a\sin t$, $y = t\cos t$, where $-\frac{\pi}{2} \leq t \leq \frac{\pi}{2}$ and a is a constant. It is given that the normal to C at $x = 0$ is parallel to the x-axis.

(i) Show that $a = 1$. [3]

(ii) Sketch C, giving the coordinates of any points of intersection with the axes. [2]

(iii) Find the area of the region enclosed by C and the x-axis. [3]

4 The functions f and g are defined as follows.

$$\mathrm{f} : x \mapsto \frac{x^2+1}{2x}, \quad x > 0,$$

$$\mathrm{g} : x \mapsto \frac{1}{x}, \quad x > 0.$$

(i) Determine whether the composite function gf exists. If it exists, define gf in a similar form and find the range of gf. [4]

(ii) Give a reason why f does not have an inverse function. [1]

(iii) If the domain of f is further restricted to $x \geq k$, state the least value of k for which the function f^{-1} exists. Find $\mathrm{f}^{-1}(x)$ and write down the domain of f^{-1}. [4]

5 **(i)** Solve the equation

$$z^6 + 64 = 0,$$

giving the roots in the form $r\mathrm{e}^{\mathrm{i}\alpha}$, where $r > 0$ and $-\pi < \alpha \le \pi$. [3]

(ii) The roots in part **(i)** represented by z_n, where $1 \le n \le 6$, are such that $-\pi < \arg(z_1) < \arg(z_2) < \cdots < \arg(z_6) \le \pi$. Show the roots on an Argand diagram and describe geometrically the relationship between the roots. [4]

The complex number w satisfies the equation $\left|\mathrm{i}w + 4 + 4\sqrt{3}\mathrm{i}\right| = 2$.

(iii) On the same Argand diagram, sketch the locus $\left|\mathrm{i}w + 4 + 4\sqrt{3}\mathrm{i}\right| = 2$. [3]

(iv) Hence, find the maximum possible value of $|w - z_n|$. [2]

Section B: Statistics [60 marks]

6 The CEO of a company with 40 000 employees wishes to investigate employees' opinions about the food stalls in the staff canteen. 2% of the employees will be chosen to take part in the survey. Explain briefly how the CEO could carry out a survey using

(i) random sampling, [2]

(ii) quota sampling. [2]

7 In a certain town, every car license plate number is a 4-digit number where the digits are chosen from 1 to 9 and cannot be repeated.
Find the number of different car license plate numbers if

(i) there are no restrictions, [1]

(ii) the digits of the car license plate number must not be in ascending order from left to right, [2]

(iii) exactly one of the digits is an even number. [2]

Due to an increasing population, it is decided that the digits used in the car license plate number can be repeated.

(iv) Find the number of different car license plate numbers where no digits can be larger than the third digit. [2]

8 **(a)** Given that events X and Y are independent, prove that events X and Y' are independent. [2]

(b) For events A and B, it is given that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.6$ and $\mathrm{P}(A' \mid B') = 0.3$. Find

(i) $\mathrm{P}(A \cap B)$, [3]

(ii) $\mathrm{P}(B' \mid A)$. [2]

Stating your reason, determine if events A and B are

(iii) mutually exclusive, [1]

(iv) independent. [1]

9 Rickie takes the train home after work on weekdays.

(i) The number of days in a week where Rickie finds a seat on the train is denoted by A. State, in context, two assumptions needed for A to be well modelled by a binomial distribution. [2]

Assume now that A has the distribution B(5, 0.65).

(ii) Rickie is contented if he finds a seat on two or three days in a week. Using a suitable approximation, find the probability that in a year (52 weeks), Rickie is contented in no more than 30 weeks. [5]

10 In a certain country, it is to be assumed that the number of drug trafficking cases per week can be modelled by the distribution Po(0.2) and the number of cigarette trafficking cases per week can be modelled by the independent distribution Po(0.7).

(i) Find the probability that, in a randomly chosen period of 8 weeks,

(a) the country has more than 6 drug trafficking cases, [2]

(b) the total number of drug and cigarette trafficking cases is fewer than 5. [2]

(ii) The probability that the country sees fewer than 2 drug trafficking cases in a period of n weeks is less than 0.01. Express this information as an inequality in n, and hence find the smallest possible integer value of n. [3]

(iii) Give two reasons in context why the assumptions made at the start of the question may not be valid. [2]

11 In this question, you should state clearly the values of the parameters of any normal distribution you use.

The masses, in grams, of towels manufactured by companies Alpha and Bravo are modelled as having independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard Deviation
Alpha	μ	20
Bravo	275	15

(i) Given that 6.68% of the towels from Alpha have mass more than 380 grams, show that the value of μ is 350 grams, correct to 3 significant figures. [2]

Towels from Alpha and Bravo are soaked in water to investigate their absorbency. A soaked towel from Alpha is 60% heavier than its dry towel, while a soaked towel from Bravo is 50% heavier than its dry towel.

(ii) Find the probability that the total mass of 4 soaked towels from Alpha and 2 soaked towels from Bravo exceeds 3 kilograms. [4]

12 A new Burger Chain, Burger Queen, claims that the mean waiting time for a burger is at most 4 minutes. The CEO of Burger Jack decides to record the waiting time, x minutes, for a burger at Burger Queen at 80 different locations. The results are summarised by

$$\sum(x-4)=25\ ,\ \sum(x-4)^2=140.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 1% level of significance, whether there is any evidence to doubt Burger Queen's claim. [4]

(iii) It is assumed that the standard deviation of the waiting time for a Burger Queen burger is 1.5 minutes. Given that the mean waiting time at another 80 locations is $\bar{x}$, use an algebraic method to find the set of values of $\bar{x}$ for which Burger Queen's claim would not be rejected at the 10% level of significance. [3]

13 **(i)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A), (B), (C) below. In each case your diagram should include 6 points, approximately equally spaced with respect to x, and with all x- and y- values positive. The letters a, b, c, d, e and f represent constants.

(A) $y=a+bx^2$, where a is positive and b is negative,

(B) $y=c+\dfrac{d}{x}$, where both c and d are positive,

(C) $y=e+fx$, where e is positive and f is negative. [3]

An archaeologist found an unknown substance on an excavation trip. Research is being carried out to investigate how the mass of the substance varies with time, measured from when it is placed in a cooled chamber. Observations at successive times give the data shown in the following table.

Time (x hours)	100	800	1500	3000	6000	8000
Mass (y grams)	25	8	5	4	3.5	3.3

(ii) Draw the scatter diagram for these values, labelling the axes. [1]

(iii) Explain which of the three cases in part **(i)** is the most appropriate for modelling these values, and calculate the product moment correlation coefficient for this case. [2]

(iv) Use the case that you identified in part **(iii)** to find the equation of a suitable regression line and estimate the time when the mass of the substance is 10 grams. Comment on the reliability of the estimate. [3]

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