MERIDIAN JUNIOR COLLEGE

JC2 Preliminary Examination

Higher 2

H2 Mathematics 9740/02

Paper 2 **21 September 2016**

3 Hours

Additional Materials: Writing paper
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 Referred to the origin O, points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors that are neither perpendicular nor parallel to each other.

The length of projection of $\mathbf{a}$ onto $\mathbf{b}$ and the length of projection of $\mathbf{b}$ onto $\mathbf{a}$ are equal. Show that $|\mathbf{a}| = |\mathbf{b}|$. [2]

Hence state the geometrical interpretation of $|\mathbf{a} \times \mathbf{b}|$. [1]

It is further given that $\mathbf{a} = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$, where $p < 0$.

(i) Find the exact value of p. [2]

(ii) A circle with centre O passes through A and B. Find the area of the minor sector OAB. [4]

2 **(a)** The roots of the equation $z^3 - z^2 - z - 15 = 0$ are denoted by z_1, z_2 and z_3 where $\arg(z_1) = 0$, and $\arg(z_2) > \arg(z_3)$. Find z_1, z_2 and z_3 and show these roots on an Argand diagram. [3]

Explain why the locus of all points z such that $|z+1| = 2$ passes through the roots represented by z_2 and z_3 Draw this locus on the same Argand diagram. [3]

(b) **(i)** Show that $1 + \mathrm{e}^{\mathrm{i}\theta} = 2\mathrm{e}^{\mathrm{i}\frac{\theta}{2}} \cos\frac{\theta}{2}$. [2]

(ii) Hence find, in trigonometric form, the imaginary part of the complex number $w = \dfrac{\mathrm{e}^{\mathrm{i}\theta}}{1+\mathrm{e}^{\mathrm{i}\theta}}$. [2]

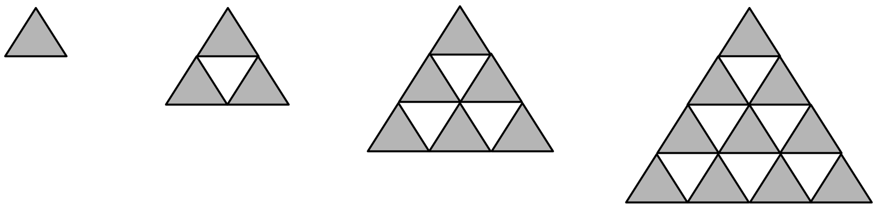
3 Newton's law of cooling states that the rate of decrease of temperature of a hot body is proportional to the difference in temperature between the body and its surroundings. Using t for time in minutes, θ for temperature of the body in °C and α for the temperature of the surroundings (assumed constant), express the law in the form of a differential equation. [1]

(i) Show that the general solution of the differential equation may be expressed in the form $\theta = \alpha + Ae^{-kt}$ where A and k are constants. [3]

(ii) Given that $\theta = 9\alpha$ when $t = 0$ and that $\theta = 5\alpha$ when $t = T$, find, in terms of T, the value of t when $\theta = 2\alpha$. [3]

(iii) State what happens to θ for large values of t and sketch the solution curve of θ against t. [3]

4 **(a)** Judith is making a pattern consisting of rows of matchstick triangles as shown. She uses three matchsticks to complete a triangle. She adds two more triangles in the second row, three more triangles in the third row and four more triangles in the fourth row.



Judith has completed $n-1$ rows in the pattern. How many matchsticks does she need in order to form the n^{th} row? [1]

Show that the total number of matchsticks used in making a pattern with n rows is $\frac{3n(n+1)}{2}$. Hence find the maximum number of complete rows she is able to make with two thousand matchsticks. [4]

(b) A geometric progression has first term a and second term b, where a and b are non-zero constants. Given that the sum to infinity of the series is $a + 2b$, find the common ratio. [2]

The sum of the first n terms is denoted by G_n. Find G_n in terms of a and n. Hence show that $\sum_{n=1}^{N} G_n = 2aN - G_N$. [4]

Section B: Statistics [60 marks]

5 **(i)** Describe what is meant by 'systematic sampling'. [2]

(ii) A bakery wishes to gather feedback on what residents in the neighbourhood think of its new salted egg lava buns. A surveyor is hired to survey a sample of 150 residents who visit the bakery during the evening rush hour using systematic sampling. State, in this context, one advantage and one disadvantage of this procedure. [2]

6 A box consists of a very large number of balls, of which 20% are red and 80% are white.

A game consists of a player drawing n balls at random from the box and counting the number of red balls drawn. If at most one red ball is drawn, the player wins. If more than two red balls are drawn, the player loses. If exactly two red balls are drawn, the player draws another n balls and if none of these n balls drawn are red, the player wins. Otherwise, the player loses.

Show that the probability that a randomly chosen player wins is P where

$$P=(0.8+0.2n)(0.8)^{n-1}+\binom{n}{2}(0.2)^2(0.8)^{2n-2}.$$ [3]

(i) Given that the probability that a randomly chosen player wins is less than 0.1, write down an inequality in terms of n to represent this information. Hence find the least possible value of n. [3]

(ii) Given instead that $P=0.3$, find the probability that out of 100 games played, at least 40 games are won. [2]

7 An overseas study revealed that school children sleep an average of 6.5 hours each night. Ms Patricia believes that the children in her school sleep even fewer than that. She took a random sample of 8 children from her school. The number of hours of sleep each child gets at night was reported as:

5.9 6 6.1 6.2 6.3 6.5 6.7 6.9

Test, at the 8% level of significance, whether this evidence supports Ms Patricia's belief, stating clearly any assumption made. [5]

Ms Patricia conducted a further study involving a random sample of 15 children from another school and the number of hours of sleep each child gets at night is recorded. The sample mean is $\bar{x}$ and the sample variance is 0.849. Find the set of values of $\bar{x}$ for which the null hypothesis would be rejected at the 8% level of significance. [3]

8 The mass of a randomly chosen bar of body soap manufactured by a factory has a normal distribution with mean 110 grams and standard deviation 1.5 grams.

(i) Find the probability that the difference in sample means between any two random samples of 20 bars of body soap each, is within 0.5 grams. [4]

(ii) Five randomly chosen bars of body soap are liquefied and separated into four equal portions, which are each placed into a bottle. Find the probability that the mass of liquid body soap in a randomly chosen bottle exceeds 140 grams. [4]

The factory ventured into the manufacturing of coconut oil soap as its new product and the mass of a randomly chosen bar of coconut oil soap has a normal distribution. A random sample of 15 bars of coconut oil soap is taken and the mass, u grams, of each bar is measured. The results are summarised by $\sum u = 1590$, $\sum u^2 = 169046$.

(iii) Find unbiased estimates of population mean and variance. [2]

9 **(a)** For events A and B, it is given that $\mathrm{P}(A)=\frac{1}{4}$ and $\mathrm{P}(B)=\frac{1}{2}$.

(i) Given that $\mathrm{P}(A'\mid B)=\frac{3}{4}$, determine whether events A and B are independent and calculate $\mathrm{P}(A\cup B)$. [3]

(ii) For a third event C, it is given that $\mathrm{P}(C\mid A)=\frac{2}{3}$. Find the value of $\mathrm{P}(A\cap C)$. [2]

(b) Find the number of ways in which the word EVERYDAY can be arranged if

(i) all the vowels (A, E) must be together and the two 'Y's must be separated, [3]

(ii) the repeated letters E and Y must appear symmetrical about the centre of the word (e.g. EVRYYDAE, YVERDEAY). [2]

10 **(i)** A bakery sells cookies in tins and keeps track of the number of tins sold per week. State two conditions under which a Poisson distribution would be a suitable probability model for the number of tins sold in a week. [2]

(ii) Two types of cookies, chocolate and raisin, are sold. The mean number of tins for chocolate cookies sold in a week is 2.4. The mean number of tins for raisin cookies sold in a week is 1.8. Use a Poisson distribution to find the probability that in a given week, the total number of tins sold is more than 9. [3]

(iii) Use a normal approximation to the Poisson distribution to find the probability that the total number of tins sold in 4 weeks is at least 15 but not more than 25. [4]

(iv) Explain why the Poisson distribution may not be a good model for the number of cookies sold in a year. [1]

11 The table gives the population y, in thousands, for a particular species of mammal over 10 years.

Year, x	1	2	3	4	5	6	7	8	9	10
Population, y (in thousands)	10.8	8.7	6.9	5.5	4.4	3.5	2.8	2.3	1.8	1.4

(i) Find the equation of the regression line of y on x, giving your answer to 3 decimal places. [1]

(ii) Let Y be the value obtained by substituting a value of x into the equation of the regression line of y on x found in **(i)**. Find $\sum(y-Y)^2$. [1]

(iii) For each of the values of x, Y' is given by $Y' = A + Bx$, where A and B are any constants. What can you say about the value of $\sum(y-Y')^2$? [1]

(iv) Draw a scatter diagram to illustrate the data. [2]

An animal conservationist suggested the model $\ln y = c + dx$ for this set of data.

(v) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between

(a) x and y,

(b) x and $\ln y$. [2]

(vi) Use your answers to parts **(iv)** and **(v)** to explain which of $y = a + bx$ or $\ln y = c + dx$ is the better model. [1]

(vii) Using the better model found in **(vi)**, predict the population of this species in the 20th year. [2]

END OF PAPER