



TAMPINES JUNIOR COLLEGE

JC2 PRELIMINARY EXAMINATION



MATHEMATICS

9740/02

Paper 2

Thursday, 15 Sep 2016

3 hours

Additional Materials: Answer paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in, including the Cover Page.

Write in dark blue or black pen on both sides of the paper.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 The curve C has parametric equations

$$x = 2\cos t, \quad y = 3\sin t.$$

- (i) Find the equation of the normal to C at the point P with parameter θ , leaving your answer in terms of θ . [3]
- (ii) This normal to C at the point P meets the x - and y -axes at points A and B respectively. Find the cartesian equation of the locus of the midpoint of AB as θ varies. [4]

2 Using partial fractions, find the exact value of

$$\int_0^1 \frac{15x^2 - x + 17}{(2x + 1)(x^2 + 4)} dx. \quad [8]$$

3 Mrs X wants to sew handmade gifts for the guests attending her daughter's wedding. On the first day of gift preparation, she spends 270 minutes of her time. Subsequently, she will decrease the amount of time spent each day on gift preparation by a certain amount. The total time taken to complete her gift preparation is 9000 minutes.

- (i) Mrs X spends, on each subsequent day, 2.5% less time on gift preparation than on the previous day. Find, to the nearest minute, the time Mrs X spends on the 10th day, and find the minimum number of days Mrs X requires to complete her gift preparation. [6]
- (ii) It takes 20 minutes for Mrs X to complete one gift. If Mrs X has the opportunity to make more gifts using this model, in theory, find the maximum number of gifts she can complete. [2]
- (iii) After doing some calculations, Mrs X realises that she has to decrease the number of days spent on gift preparation. She still spends 270 minutes on the first day, but on each subsequent day, the amount of time spent is 4 minutes less than on the previous day. Find the minimum number of days Mrs X requires to complete her gift preparation. [3]

4 (a) The complex number w is given by $3 + 3(\sqrt{3})i$.

(i) Find the modulus and argument of w , giving your answers in exact form. [2]

(ii) Without using a calculator, find the smallest positive integer value of n for which

$$\left(\frac{w^3}{w^*} \right)^n \text{ is a real number.} \quad [4]$$

(b) The complex number z is such that $z^5 = -4\sqrt{2}$.

(i) Find the values of z in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [4]

(ii) Show the roots on an Argand diagram. [2]

(iii) The roots represented by z_1 and z_2 are such that $0 < \arg(z_1) < \arg(z_2) < \pi$. The locus of all points z such that $|z - z_1| = |z - z_2|$ intersects the line segment joining points representing z_1 and z_2 at the point P . P represents the complex number p . Find, in exact form, the modulus and argument of p . [2]

Section B: Statistics [60 marks]

5 It is desired to conduct a survey among university students regarding the use of the university's facilities. A random sample of 100 students is obtained.

(i) Explain what is meant in this context by the term 'a random sample'. [2]

(ii) Now it is necessary to obtain a representative range by faculties. Name a more appropriate sampling method and explain how it can be carried out. [2]

6 75% of the employees in a factory own a cell phone.

(i) A random sample of 8 employees is taken. Find the probability that the number of employees who own a cell phone is between 4 and 6 inclusive. [2]

In an industrial park, every factory has 160 employees.

(ii) Use a suitable approximation to find the probability that in a randomly selected factory, at least 115 employees own a cell phone. [3]

(iii) A random sample of 15 factories in the industrial park is taken. Find the probability that at most 11 of these factories each have at least 115 employees who own a cell phone. [2]

- 7 The average number of parking tickets that a traffic warden issues per day is being investigated.

(i) State, in context, two assumptions that need to be made for the number of parking tickets issued per day to be well modelled by a Poisson distribution. [2]

Assume that the number of parking tickets issued per day has the distribution $Po(3.6)$.

(ii) Find the probability that, in seven days, the traffic warden issues more than 22 parking tickets altogether. [3]

(iii) The probability that the traffic warden issues more than N parking tickets altogether in 10 days is less than 0.05. Using a suitable approximation, find the least possible value of N . [2]

- 8 A fruit stall sells papayas. The mass of papayas is denoted by X kg. The stall owner claims that the mean mass of the papayas is at least 1.2 kg. The masses of a random sample of 8 papayas are summarised by

$$\sum x = 8.84, \quad \sum x^2 = 9.95.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Stating a necessary assumption, test at the 5% level of significance whether there is any evidence to doubt the stall owner's claim. [5]

- 9 Two players A and B decide to play two consecutive card games. A fair coin is tossed to decide which player has the first move in the first game. The loser of the first game has the first move in the second game. A player must win both games to be declared the overall winner. If each player wins a game, the result is a draw.

When A has the first move in a game, the probability that A wins that game is $\frac{2}{3}$. When B has

the first move in a game, the probability that B wins that game is $\frac{3}{5}$. Every game ends with

either A or B as the winner.

(i) Show that the probability that two consecutive card games end in a draw is $\frac{142}{225}$. [2]

(ii) Given that A wins the second game, find the probability that two consecutive card games end in a draw. [3]

(iii) To make their games more enjoyable, A and B agree to change the procedure for deciding who has the first move in the first game. As a result of their new procedure, the probability of A having the first move in the first game is p . Find the exact value of p which gives A and B equal chances of winning both games. [3]

10(a) Sketch a scatter diagram that might be expected when x and y are related approximately by $y = px^2 + q$ in each of the cases (i) and (ii) below. In each case your diagram should include 5 points, approximately equally spaced with respect to x , and with all x - and y -values positive.

- (i) p is positive and q is positive,
- (ii) p is negative and q is positive. [2]

(b) A car website gives the following information on the ages in months (m) and resale price in dollars (P) of used passenger cars of a particular model.

m	10	20	28	35	40	45	56	62	70	74
P	110600	79900	78700	69200	66100	60200	53800	50600	46700	43800

It is thought that the price after m months can be modelled by one of the formulae

$$P = am + b, \quad P = c \ln m + d,$$

where a , b , c and d are constants.

- (i) Find, correct to 4 decimal places, the value of the product moment correlation coefficient between
 - (A) m and P ,
 - (B) $\ln m$ and P . [2]
- (ii) Explain which of $P = am + b$ and $P = c \ln m + d$ is the better model and find the equation of a suitable line for this model. [3]
- (iii) Use the equation of your regression line to estimate the price of a used passenger car that is 80 months old. Comment on the reliability of your estimate. [2]

11 A group of 5 girls and 7 boys play ice breaker games to get to know each other.

- (i) The group stands in a line.
 - (a) Find the number of different possible arrangements. [1]
 - (b) The girls have names that start with different letters. Find the number of different possible arrangements in which all the girls are separated, with the girls' names in alphabetical order. [2]
- (ii) The group forms two circles of 6, with one circle inside the other, such that each person in the inner circle stands facing a person in the outer circle. Find the number of different possible arrangements. [2]
- (iii) The group has to split into a group of 3, a group of 4, and a group of 5. Find the number of possible ways in which the groups can be chosen if there is no girl in at least one of the groups. [4]

- 12 A supermarket sells two types of guava, *A* and *B*. The masses, in grams, of the guava of each type have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean (g)	Standard deviation (g)
Type <i>A</i>	200	12
Type <i>B</i>	175	12

Find the probability that

- (i) the total mass of 4 randomly chosen guava of type *A* is more than 810 g, [2]
(ii) the mean mass of 4 randomly chosen guava of type *A* differs from the mean mass of 3 randomly chosen guava of type *B* by at least 30 g. [3]

Mr Tan buys 20 guavas, m of them are guava of type *A* and the rest are guava of type *B*.

- (iii) Find the least value of m such that the probability that the total mass of these 20 guavas exceeding 3500 g is more than 0.95. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [4]

End of Paper