TEMASEK JUNIOR COLLEGE, SINGAPORE

Preliminary Examination 2016
Higher 2

MATHEMATICS — 9740/01

Paper 1

30 August 2016

Additional Materials: Answer paper
List of Formulae (MF15)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages.

TEMASEK JUNIOR COLLEGE, SINGAPORE
PASSION PURPOSE DRIVE

1 A fitness assessment walk is conducted where participants walk briskly around a running path. The participants' walking time and heart rate are recorded at the end of the walk.

The formula for calculating the Fitness Index of a participant is as follows:

420 + (Age × 0.2) – (Walking Time × a) – (Body Mass Index × b) – (Heart Rate × c)

where a, b and c are real constants.

Data from 3 participants, Anand, Beng and Charlie are given in the table.

Name	Age	Walking Time	Body Mass Index	Heart Rate	Fitness Index
Anand	32	17.5	25	100	102.4
Beng	19	18.5	19	120	92.6
Charlie	43	17	23	90	121.2

Find the values of a, b and c. [3]

2 Solve the inequality $\dfrac{x^2 - 2a^2}{x} < a$, giving your answer in terms of a, where a is a positive real constant. [3]

Hence solve $\dfrac{x^2 - 2a^2}{|x|} < a$. [2]

3 **(i)** Use the substitution $u = x^2$ to find $\displaystyle\int \frac{x}{\sqrt{k^2 - x^2}}\,dx$ in terms of x and the constant k. [3]

(ii) Find the exact value of $\displaystyle\int_0^2 f(x)\,dx$, where

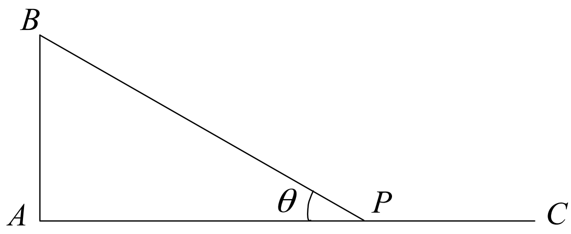
$$f(x) = \begin{cases} \dfrac{2}{6 - x^2}, & 0 \le x < \sqrt{2}, \\ \dfrac{x}{\sqrt{6 - x^2}}, & \sqrt{2} \le x < 2. \end{cases}$$ [3]

4 Relative to the origin O, the points A, B, M and N have position vectors **a, b**, **m** and **n** respectively, where **a** and **b** are non-parallel vectors. It is given that $\mathbf{m} = \lambda\mathbf{a} + (1 - \lambda)\mathbf{b}$ and $\mathbf{n} = 2(1 - \lambda)\mathbf{a} - \lambda\mathbf{b}$ where λ is a real parameter.

Show that $\mathbf{m} \times \mathbf{n} = (3\lambda^2 - 4\lambda + 2)\,(\mathbf{b} \times \mathbf{a})$. [2]

It is given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 4$ and the angle between **a** and **b** is $\dfrac{\pi}{6}$. Find the smallest area of the triangle MON as λ varies. [4]

5



In the diagram, A and C are fixed points 500 m apart on horizontal ground. Initially, a drone is at point A and an observer is standing at point C. The drone starts to ascend vertically at a steady rate of 3 m s^{-1} as the observer starts to walk towards A with a steady speed of 4 m s^{-1}. At time t, the drone is at point B and the observer is at point P.

Given that the angle APB is θ radians, show that $\theta = \tan^{-1}\left(\dfrac{3t}{500-4t}\right)$. [2]

(i) Find $\dfrac{\mathrm{d}\theta}{\mathrm{d}t}$ in terms of t. [2]

(ii) Using differentiation, find the time t when the rate of change of θ is maximum. [4]

6 The functions f and g are defined by

$$\mathrm{f}: x \mapsto \ln\left(x^2 - x + 1\right),\ x \in \mathbb{R},\ x \le 1,$$
$$\mathrm{g}: x \mapsto \mathrm{e}^x,\ x \in \mathbb{R}.$$

Sketch the graph of f and explain why f does not have an inverse. [2]

The function h is defined by

$$\mathrm{h}: x \mapsto \mathrm{f}(x),\ x \in \mathbb{R},\ x \le k.$$

State the maximum value of k such that h^{-1} exists. [1]

Using this maximum value of k,

(i) show that the composite function gh exists, [1]

(ii) find $(\mathrm{gh})^{-1}(x)$, stating the domain of $(\mathrm{gh})^{-1}$. [4]

7 **(a)** The positive integers are grouped into sets as shown below, so that the number of integers in each set after the first set is three more than that in the previous set.

$$\{1\}, \{2, 3, 4, 5\}, \{6, 7, 8, 9, 10, 11, 12\}, \ldots$$

Find, in terms of r, the number of integers in the rth set. [1]

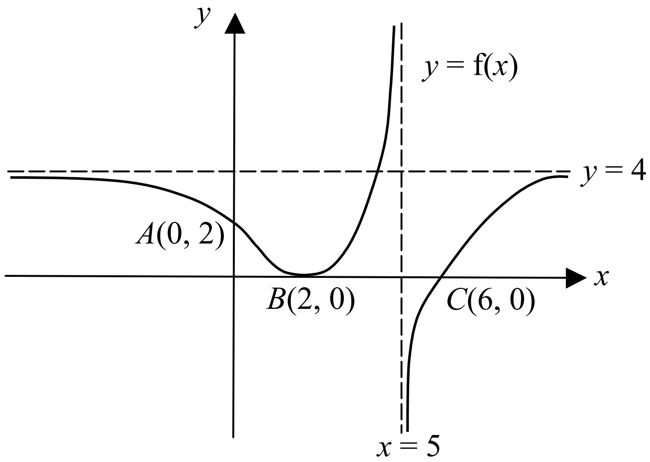
Show that the last integer in the rth set is $\dfrac{r}{2}(3r-1)$. [2]

Deduce, in terms of r, the first integer in the rth set. [2]

(b) Find $\displaystyle\sum_{r=1}^{n}\left(1+\frac{1}{2}+\left(\frac{1}{2}\right)^2+\ldots+\left(\frac{1}{2}\right)^r\right)$ in terms of n. [4]

[Turn over

8 The graph of $y = \mathrm{f}(x)$ intersects the axes at $A(0, 2)$, $B(2, 0)$ and $C(6, 0)$ as shown below. The lines $y = 4$ and $x = 5$ are asymptotes to the graph, and $B(2, 0)$ is a minimum point.



On separate diagrams, sketch the graphs of

(i) $y = \mathrm{f}(|x|)$, [2]

(ii) $y^2 = \mathrm{f}(x)$, [3]

(iii) $y = \dfrac{1}{\mathrm{f}(x)}$, [3]

stating the equations of any asymptotes, coordinates of any stationary points and points of intersection with the axes.

9 **(a)** Given that x is small such that x^3 and higher powers of x can be neglected, show that

$$\frac{\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)}{\sqrt{2-\cos x}} \approx a + bx + cx^2,$$

for constants a, b and c to be determined. [4]

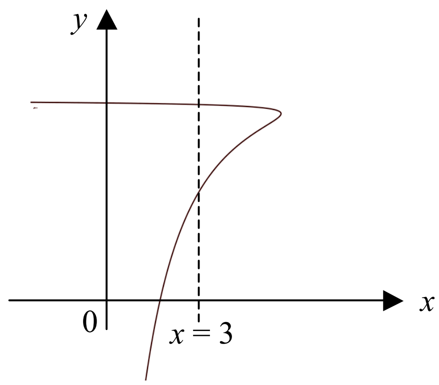
(b) The curve $y = \mathrm{f}(x)$ passes through the point $(0, -1)$ and satisfies the differential equation

$$(1+x^2)\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^{-y}.$$

(i) Find the Maclaurin series for y, up to and including the term in x^2. [3]

(ii) By using an appropriate expansion from the List of Formulae (MF15), obtain the Maclaurin series for $\ln(2+y)$, up to and including the term in x^2. [3]

10



The diagram shows the curve with parametric equations

$$x = 2t + t^2, \quad y = \frac{1}{(1-t)^2}, \quad \text{for } t < 1.$$

The curve has a vertical asymptote $x = 3$.

(i) Find the coordinates of the points where the curve cuts the y-axis. [2]

(ii) Find the equation of the tangent to the curve that is parallel to the y-axis. [4]

(iii) Express the area of the finite region bounded by the curve and the y-axis in the form $\int_a^b \mathrm{f}(t)\,\mathrm{d}t$, where a, b and f are to be determined. Use the substitution $u = 1 - t$ to find this area, leaving your answer in exact form. [5]

11 On the remote island of Squirro, ecologists introduced a non-native species of insects that can feed on weeds that are killing crops. Based on past studies, ecologists have observed that the birth rate of the insects is proportional to the number of insects, and the death rate is proportional to the square of the number of insects. Let x be the number of insects (in hundreds) on the island at time t months after the insects were first introduced.

Initially, 10 insects were released on the island. When the number of insects is 50, it is changing at a rate that is $\frac{3}{4}$ times of the rate when the number of insects is 100. Show that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \beta x(2 - x)$$

where β is a positive real constant. [3]

Solve the differential equation and express x in the form $\frac{p}{1 + q\mathrm{e}^{-2\beta t}}$, where p and q are constants to be determined. [6]

Sketch the solution curve and state the number of insects on the island in the long run. [3]

[Turn over

12 **(a)** The complex numbers z_1 and z_2 satisfy the following simultaneous equations

$$2z_1 + \mathrm{i}\,z_2^* = 7 - 6\mathrm{i},$$

$$z_1 - \mathrm{i}\,z_2 = 6 - 6\mathrm{i}.$$

Find z_1 and z_2 in the form $x + y\mathrm{i}$, where x and y are real. [4]

(b) It is given that $w = \frac{1}{2} - \frac{1}{2}\mathrm{i}$. Find the modulus and argument of w, leaving your answers in exact form. [2]

It is also given that the modulus and argument of another complex number v is 2 and $\frac{\pi}{6}$ respectively.

(i) Find the exact values of the modulus and argument of $\frac{v}{w^*}$. [3]

(ii) By first expressing v in the form $\sqrt{c} + d\,\mathrm{i}$ where c and d are integers, find the real and imaginary parts of $\frac{v}{w^*}$ in surd form. [3]

(iii) Deduce that $\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$. [2]

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