SAINT ANDREW'S JUNIOR COLLEGE

Preliminary Examination

MATHEMATICS
Higher 2 **9740/02**

Paper 2

Thursday **15 September 2016** **3 hours**

Additional materials : Answer paper
List of Formulae (MF15)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, civics group and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.

Answer **all** the questions. Total marks : **100**

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of 7 printed pages including this page.

Section A: Pure Mathematics [40 marks]

1 Prove by the method of mathematical induction that $\sum_{r=2}^{n}(r-1)\ln\left(\frac{r}{r-1}\right)=\ln\left[\frac{n^{n-1}}{(n-1)!}\right]$. **[5]**

2 Solve the inequality $\frac{x^2+6x+8}{x-1}\geq 0$.

Hence, by completing the square, solve the inequality $\frac{y^2+2y+15}{|y+1|-1}\geq -6$. **[6]**

3 It is given that $\mathrm{f}(x)=\frac{5-ax^2}{1+x^2}$ where $a>1$, $a\in$°$^{+}$.

(i)

Sk

etch $y=\mathrm{f}(x)$, showing clearly the coordinates of the turning point, any intersections with the axes and the equation(s) of any asymptote(s). **[3]**

(ii) By drawing a sketch of another suitable curve on the same diagram, find the number of real roots of the equation

$$x^4+(a+1)x^2-5=0 \text{ .}$$ **[2]**

(iii) Let $\mathrm{g}(x)=x^4+(a+1)x^2-5$. Show that $\mathrm{g}(x)=\mathrm{g}(-x)$. What can be said about the four roots of the equation $\mathrm{g}(x)=0$? **[3]**

4 A curve C has parametric equations

$$x=a\sin 2t\text{ , }y=a\sin 3t$$

where $0\leq t\leq\frac{\pi}{2}$and a is a positive constant.

(i) Find the gradient of C at the point $(a\sin 2\theta\text{ , }a\sin 3\theta)$ where $0\leq\theta\leq\frac{\pi}{2}$. Hence, what can be said about the tangent to C as $\theta\to\frac{\pi}{4}$? **[3]**

(ii) Find the equation of the normal, in exact form, at the point where $t=\frac{\pi}{12}$. **[3]**

(iii) With the aid of a sketch, show that the area bounded by the curve C, the y-axis and the line $y=\frac{a\sqrt{2}}{2}$ can be written as

$$3a^2\int_0^{\frac{\pi}{12}}\left(\cos 3t\sin 2t\right)\mathrm{d}t\ .$$

Hence, find the exact area of the region bounded by the curve C, the y-axis and the normal to the curve at $t=\frac{\pi}{12}$ in the form $ka^2\left[b\cos\frac{\pi}{12}+c\sin\frac{\pi}{12}\right]+da^2$, where k, b, c and d are constants to be determined. **[7]**

5 The complex number z is given by $z=re^{i\theta}$, where $1\le r\le 2$ and $\frac{1}{6}\pi\le\theta\le\frac{3}{4}\pi$.

(i) State $|z|$ and arg(z) in terms of r and θ. Hence, draw an Argand diagram to show the locus of z as r and θ varies. You should identify the modulus and argument of the end-points of the locus. **[3]**

(ii) Find the exact minimum value of $|z+5-6\mathrm{i}|$ and the corresponding complex number z representing the point at which this minimum value occurs, giving your answer in the form x + iy, where x and y are real numbers. **[3]**

Another complex number w satisfies the equation $\arg(w-2\sqrt{3})=\frac{5\pi}{6}$.

(iii) On the same diagram as part **(i)**, sketch the locus of w and indicate the set of points that satisfies both the locus of z and w. **[2]**

Section B: Statistics [60 marks]

6 A survey is to be carried out to obtain feedback from the members of a new female-only fitness club regarding its various facilities and fitness classes. The membership of this fitness club comprises 5000 female members and the number of members belonging to the various age groups are given in the table below:

Age group	18 - 25	26-30	31 – 40	41 – 50	51 & above
Number of members	500	1000	1500	1500	500

It is proposed to carry out the survey by interviewing members who visit the club on a particular weekday in the morning.

(i) Explain why this proposed method is inappropriate. **[1]**

(ii) Suggest an appropriate method of carrying out the survey and describe how you intend to implement the sampling method to obtain a representative sample of 200 members. **[3]**

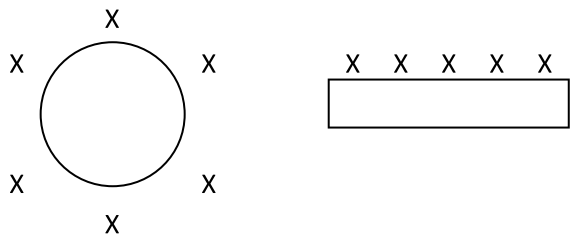
7 **(a)** For events X and Y, it is given that $P(X \cup Y) = \frac{5}{8}, P(X \cap Y') = \frac{7}{24}$ and $P(X' | Y) = \frac{9}{16}$.

(i) Find $P(X' \cap Y)$, **[3]**

(ii) Find $P(X)$ and determine if the events X and Y' are independent. **[3]**

(b) The Mathematics department consists of 5 female teachers and 6 male teachers. After a meeting, the department went to a nearby food court for lunch. Due to the lunch crowd, they only managed to find a circular table for 6 and a long table with a row of 5 seats as shown below. The seats at both tables are fixed and cannot be rearranged.

Ms Koh was among the Mathematics teachers who attended the lunch.



(i) Find the probability that Ms Koh is seated between 2 male teachers. **[4]**

(ii) Given that Ms Koh is seated between 2 male teachers, find the probability that the male and female teachers alternate at both tables. **[3]**

8 **(i)** Given that $X \sim N(\mu, \sigma^2)$ and $P(X < 28.4) = P(X > 77.6) = 0.012$, find the value of μ and σ. **[3]**

(ii) The mass, in grams, of a randomly chosen packet of sweets is normally distributed with mean μ and variance σ^2 obtained from part **(i)**. Every packet of sweets is priced at \$1.20 per 100 grams. Find the probability that the sum of 4 packets of sweets cost at most \$2.60. **[3]**

9 The hens on a farm lay either white or brown eggs. The eggs are randomly put into boxes of six. The farmer claims that the number of brown eggs in a box can be modelled by a binomial distribution $\mathrm{B}(6, p)$.

(i) State, in context, two assumptions to support the farmer's claim. **[2]**

(ii) Given that the probability a box contains at least 5 brown eggs is 0.04096, find the value of p. **[2]**

A supermarket orders 100 boxes of eggs daily.

(iii) By using a suitable approximation, find the probability that there are at least 90 boxes that contains at most 4 brown eggs in a particular day. **[3]**

(iv) The supermarket places a daily order of 100 boxes of eggs for 8 weeks. Estimate the probability that the mean number of boxes that contains at least 5 brown eggs in a day is between 4 and 7.
[You may assume that there are 7 days in a week.] **[3]**

10 A researcher wishes to investigate the length of time that patients spend with a doctor at a particular clinic. The time a patient spends with the doctor is denoted by X minutes. Based on past records, the clinic claims that the mean length of time for the doctor to see a patient is at most 10 minutes. To test this claim, the researcher recorded the actual times spent by the doctor to see a random sample of 12 patients.

$$\sum x = 147, \quad \sum x^2 = 1927.91$$

(i) Stating a necessary assumption, carry out an appropriate test, at the 5% significance level, to determine whether there is any evidence to doubt the clinic's claim. **[5]**

(ii) Suppose now that the population standard deviation of X is 15 and that the assumption made in part **(i)** is still valid. A new sample of n patients is obtained and the sample mean length of time is found to be unchanged. Using this sample, the researcher conducts another test and found that the null hypothesis is not rejected at the 5% significance level. Obtain an inequality involving n and find the set of values that n can take. **[3]**

11 An experiment is conducted to calibrate an anemometer*. In this calibration process, the wind speed X is fixed precisely and the resulting anemometer speed Y is recorded.

For a particular anemometer, this process produced the following set of measurements:

Wind speed (m/s), X	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0
Anemometer (revs/min), Y	24	28	47	92	164	236	312	360

(i) Calculate the product moment correlation coefficient between X and Y. **[1]**

(ii) Sketch a scatter diagram for the data. **[1]**
Explain why it is advisable to sketch the scatter diagram in addition to calculating the product moment correlation coefficient before interpreting this set of bivariate data. **[1]**

A proposed model for the above data is $Y = a + bX^2$.

(iii) Calculate the product moment correlation coefficient and the equation of the least squares regression line for the proposed model.
Explain whether the model for Y on X^2 or Y on X is a better model for the data set. **[3]**

(iv) Use an appropriate regression line to estimate the value of X when the value of Y is 120. Give a reason for the choice of your regression line. **[2]**

[* *An anemometer is a device commonly used in a weather station for measuring wind speed.*]

12 The managers of 2 branches of a travel agency were discussing whether the number of customers who bought the Luxury Cruise Package per week could be modelled by a Poisson distribution. One of the managers said, "It must be assumed that the number of customers who bought the package per week is a constant."

(i) Give a corrected version of the manager's statement, and explain why the correction is necessary. **[1]**

It is given that the number of customers who bought the Luxury Cruise Package per week can be modelled by a Poisson distribution. The average number of customers who bought the Luxury Cruise Package per week at Branch A and Branch B is 3.5 and 4.5 respectively. Assume that the number of customers who bought the package at Branch A and Branch B are independent.

(ii) Find the probability that, in a randomly chosen week, the total number of customers who bought the Luxury Package at both branches is between 5 and 10. **[2]**

(iii) Given that the probability that at most one customer bought the Luxury Cruise Package at Branch A in n weeks is less than 0.1, find the value of the least n. **[3]**

(iv) Using a suitable approximation, find the probability that, in one month, the number of customers who bought the Luxury Cruise Package at Branch B exceeds the number of customers at Branch A by not more than 5. **[4]**
[You may assume that there are 4 weeks in 1 month.]

Explain why the Poisson distribution may not be a good model for the number of customers who bought the Luxury Cruise Package in a year. **[1]**

End of Paper